SPACE-TIME ADAPTIVE FIR FILTERING WITH STAGGERED PRI

R. Klemm

FGAN-FHR, 53343 Wachtberg, Germany
Tel: ++49 228 9435 377, Fax: ++49 228 9435 618, E-mail: r.klemm@fgan.de

ABSTRACT

Space-time least squares FIR filters have proven excellent clutter rejection performance at extremely low computational load so that ground moving target indication (GMTI) kann be carried out in real-time. Staggering the pulse repetition interval (PRI) is an appropriate way of avoiding Doppler ambiguities and blind velocities. Fully adaptive space-time processors can cope well with staggered echo data. FIR filtering techniques are based on constant PRI and, therefore, will suffer some degradation if the radar pulses are staggered. In this contribution the concept of re-adaptation of the FIR filter coefficients at each PRI is put forward. It is shown by simulations that the total loss caused by staggering the PRI is of the order of magnitude of a few dB. However, applying a constant FIR filter to staggered data results in dramatic losses in signal-to-clutter+noise ratio.

1. INTRODUCTION

1.1. Preliminaries

The motion of an air- or spaceborne radar causes clutter returns from the ground to be Doppler shifted. The Doppler shift of an arrival from a single scatterer is proportional to the cosine of the angle of arrival of the backscattered echo. The total of all clutter arrivals results in a Doppler broadband signal where the Doppler bandwidth is determined by the platform velocity. The clutter bandwidth degrades the detectability of slow moving targets. Space-time adaptive processing (STAP) has been shown to compensate for the platform motion effect so that basically no losses in slow moving target detection will occur.

The basis of STAP techniques is the likelihood ratio (LR) test which states that the space-time echoes received by a coherent array antenna have to be multiplied with the inverse of the space-time clutter covariance matrix, followed by coherent signal integration using a beamformer and Doppler filters. If the number of array elements N and the number of processed echoes M is large the matrix inverse may not be available by various reasons:

- Adaptation means estimation of the clutter covariance matrix. The number of operations required may be prohibitive if N and M are large.
- There may be a lack of representative clutter data to estimate the covariance matrix.
- The computation of the matrix inverse may be impossible because of extensive computational load.
- The computation of the matrix inverse may be impossible due to limited numerical accuracy.
- Implementation of the LR processor requires that all elements of the array antenna are cascaded with digitized channels. This is currently unrealistic by reasons of cost.

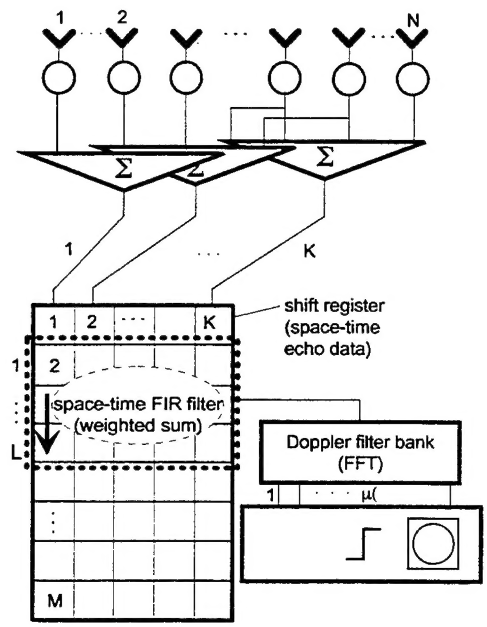

Figure 1: Overlapping subarray processor with space-time FIR filter

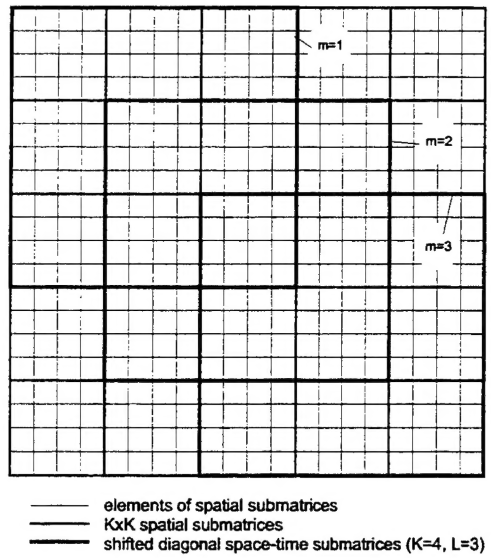

Figure 2: Matrix scheme for space-time FIR filtering, K=4, M=5

1.2. Subspace Techniques

A lot of publications deal with suboptimum approximations (frequently referred to as subspace techniques) of the space-time LR processor (e.g. WARD [11], KLEMM [2] GOLDSTEIN & REED [4]).

There are rank reducing techniques which conduct clutter suppression in the clutter subspace of the space-time covariance matrix while maintaining the order of the order of the filter matrix. The eigencanceler type of architectures (HAIMOVICH & BAR-NESS [3]) belong to this class. Saving of operations is achieved during the adaptation and filter calculation phase, however not during filtering the echo data at range sample speed.

Order reduction STAP techniques lead to reduced size architectures which promise a reduction of the computational load for adaptation, filter calculation and filterings as well. This class of processors has specific aptitude to real-time processing.

A large class of order reduction architectures are based on certain linear transforms. There are space-time transforms, spatial transforms and temporal transforms ([2, Chapter 5-7]). For large M post-Doppler techniques which operate in the Doppler domain may lead to very efficient receiver schemes.

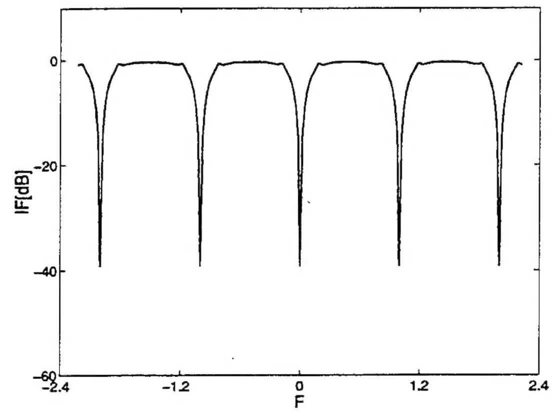

Figure 3: Fully adaptive processing, constant PRI

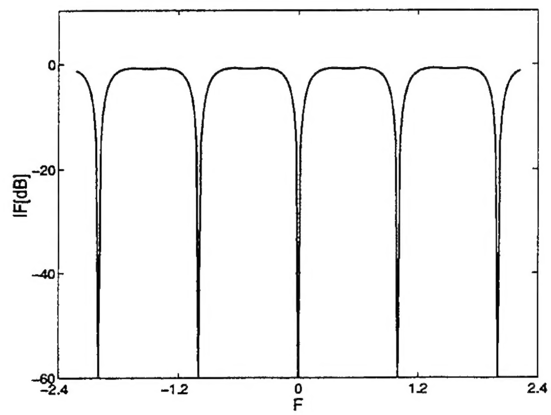

Figure 4: FIR filter, based on data from first 5 echoes, constant PRI

1.3. The Space-Time FIR Filter

1.3.1. The principle

Space-time FIR filters exploit the stationarity of echo sequences. KLEMM & ENDER [8] analysed a least squares space-time filter for GMTI processing in real-time. Related approaches have been described by BARANOSKI [10], ROMAN et al. [9] and SWINDLEHURST & PARKER [7]. In the concept of GOLDSTEIN & REED [5] several 1-delay subfilters are cascaded.

The space-time least squares FIR filter introduced by The use of *space-time* least squares FIR filters for airborne applications introduced in [8] and described in detail in [2, Chapter 7] has proven to be a highly efficient way of adaptive ground clutter suppression for moving radar. The filter is closely related to the Maximum Entropy Method (BURG [1]). A block

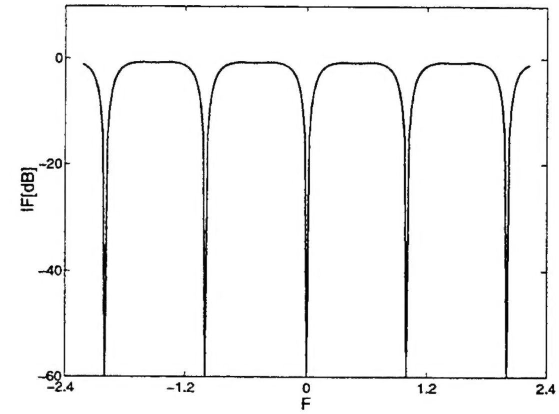

Figure 5: FIR filter, sliding calculation of coefficients, constant PRI

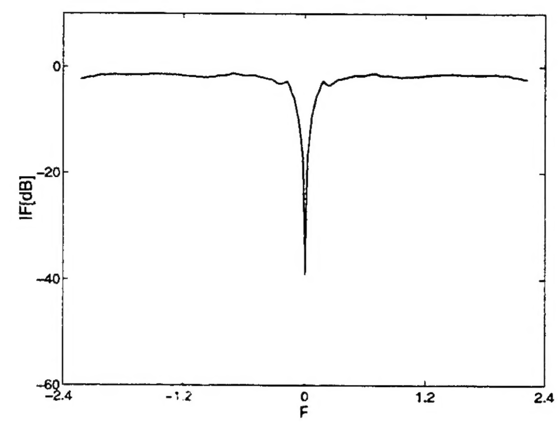

Figure 6: Fully adaptive processing, randomly staggered PRI

diagram of a FIR filter based GMTI processor is shown in Figure 1. Notice that the spatial dimension has been reduced by subdividing the array antenna into K subarrays. If $K \ll N$ the number of operations for clutter suppression is strongly reduced. Further reduction can be obtained by choosing a space-time FIR filter with $L \ll M$ delays. The FIR filter is calculated as follows:

- Choose a segment of L echoes with $L \ll M$.
- Calculate the associated space-time clutter covariance matrix. It will be one of the submatrices along the diagonal of the matrix scheme shown in Figure 2. These submatrices are denoted as $m = 1, 2, 3$.
- Calculate the inverse of the submatrix.
- Select the first $K \times KL$ block row of the inverse to become $\tilde{\mathbf{K}}$.
- Multiplying a $N \times 1$ beamformer vector $\mathbf{b}$ with $\tilde{\mathbf{K}}$ results in a $1 \times KL$ vector of space-time filter coefficients $\tilde{\mathbf{K}}\mathbf{b}$.

It has been demonstrated that the temporal filter length can be chosen independently of the coherent processing interval M (CPI). This is a desirable property, particularly when the filter is used in a multi-mode radar where the CPI varies with different operational modes. Even with very low filter dimensions (e.g., $K = L = 3$, total number of coefficients: 9) excellent approximation of the performance of the optimum processor can be achieved.

1.3.2. Mathematical description

The first column of the inverse of a Toeplitz matrix is called a *prediction error filter*. It has the property of minimizing the output power of a stationary process determined by the Toeplitz covariance matrix. The inverse of the space-time covariance matrix $\mathbf{Q}$ has the same form as $\mathbf{Q}$:

$$\mathbf{K} = \mathbf{Q}^{-1} = \begin{pmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} & \cdot & \cdot & \cdot & \mathbf{K}_{1L} \\ \mathbf{K}_{21} & \mathbf{K}_{22} & \cdot & \cdot & \cdot & \mathbf{K}_{2M} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \mathbf{K}_{M1} & \mathbf{K}_{M2} & \cdot & \cdot & \cdot & \mathbf{K}_{MM} \end{pmatrix} \quad (1)$$

Let us consider now a small segment of L echoes out of M where by L we denote the temporal filter length. We assume that $L \ll M$. Recall that the submatrices $\mathbf{K}_{ik}$ are spatial, that means, they are related either the antenna array ($N \times N$) or the subspace given by the antenna channels ($K \times K$).

Then the order reduced space-time covariance matrix becomes

$$\mathbf{K} = \mathbf{Q}^{-1} = \begin{pmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} & \cdot & \cdot & \cdot & \mathbf{K}_{1L} \\ \mathbf{K}_{21} & \mathbf{K}_{22} & \cdot & \cdot & \cdot & \mathbf{K}_{2L} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \mathbf{K}_{L1} & \mathbf{K}_{L2} & \cdot & \cdot & \cdot & \mathbf{K}_{LL} \end{pmatrix} \quad (2)$$

where L is the temporal dimension of the space-time FIR filter. Assuming for instance $K = 3$ and $L = 3$ this matrix has the dimensions 9×9. The space-time prediction error filter is the first block column of $\mathbf{K}$

$$\tilde{\mathbf{K}} = \begin{pmatrix} \mathbf{K}_{11} \\ \mathbf{K}_{21} \\ \cdot \\ \cdot \\ \mathbf{K}_{L1} \end{pmatrix} \quad (3)$$

The FIR filter operation can be formulated as follows

$$\mathbf{y} = \tilde{\mathbf{H}}^* \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{x}_M \end{pmatrix} \tag{4}$$

where

$$\tilde{\mathbf{H}}^* = \begin{pmatrix} \mathbf{K}_{11}^* & \dots & \mathbf{K}_{L1}^* & \mathbf{0}^* & \dots & & & \\ \mathbf{0}^* & \mathbf{K}_{11}^* & \dots & \mathbf{K}_{L1}^* & \mathbf{0}^* & \dots & & \\ & \mathbf{0}^* & \mathbf{K}_{11}^* & \dots & \mathbf{K}_{L1}^* & \mathbf{0}^* & \dots & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \tag{5}$$

is a shift operator, $\mathbf{0}$ is a $K \times K$ zero matrix. The spatial dimension of the FIR filter can be removed by pre-beamforming

$$\mathbf{h} = \tilde{\mathbf{K}}\mathbf{b} \tag{6}$$

so that the filtering operation can be written as

$$\mathbf{z}^* = \mathbf{H}^*\mathbf{x} = \mathbf{H}^* \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{x}_M \end{pmatrix} \tag{7}$$

where

$$\mathbf{H}^* = \begin{pmatrix} \mathbf{h}_1^* & \dots & \mathbf{h}_L^* & \mathbf{o}^* & \dots & & & \\ \mathbf{o}^* & \mathbf{h}_1^* & \dots & \mathbf{h}_L^* & \mathbf{o}^* & \dots & & \\ & \mathbf{o}^* & \mathbf{h}_1^* & \dots & \mathbf{h}_L^* & \mathbf{o}^* & \dots & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \tag{8}$$

with $\mathbf{o}$ being a K-dimensional zero vector and $\mathbf{x}_m$ is the signal vector at the array outputs at time m. Notice that $\mathbf{z}$ is temporal only with the dimension $M - L + 1$ while the dimension of the space-time vector $\mathbf{y}$ was $(M - L + 1) \times K$. The processor is completed with a Doppler filter bank with Doppler filters of length $M - L + 1$ whose output signals are obtained as

$$\mathbf{d} = \mathbf{F}\mathbf{z} \tag{9}$$

where the matrix $\mathbf{F}$ describes the Doppler filter bank, for instance, the DFT. The elements of $\mathbf{d}$ are fed into a detection device.

The improvement factor in SCNR becomes

$$IF(\omega_d) = \frac{\mathbf{s}^*(\omega_d)\mathbf{H}\mathbf{H}^*\mathbf{s}(\omega_d)\mathbf{s}^*(\omega_d)\mathbf{H}\mathbf{H}^*\mathbf{s}(\omega_d) \cdot tr(\mathbf{Q})}{\mathbf{s}^*(\omega_d)\mathbf{H}\mathbf{H}^*\mathbf{Q}\mathbf{H}\mathbf{H}^*\mathbf{s}(\omega_d) \cdot \mathbf{s}^*(\omega_d)\mathbf{s}(\omega_d)} \tag{10}$$

where we made the usual assumption that the processor is perfectly matched to the target signal vector and ω_d means Doppler frequency. In the discussion below we use the $IF(\omega_d)$ to judge the efficiency of processing and the effect of parameters. Instead of $IF(\omega_d)$ we show $IF(\mathsf{F})$ where $\mathsf{F} = \omega_d/(2\pi PRF)$ is the normalized Doppler frequency.

2. STAGGERED PRI RADAR

2.1. General Aspects

The PRF is commonly chosen constant which, however, has a couple of drawbacks

- The target Doppler cannot be estimated unambiguously.
- The clutter filter produces ambiguous notches at the blind velocities.
- The PRF can estimated by hostile ESM (electronic support measure) and countered with spot jamming.

Alternatively one may either stagger the PRF or the PRI. PRF staggering has the disadvantage that several pulse bursts have to be transmitted which means a waste of radar energy. This problem is circumvented by PRI staggering (it has the little drawback that the FFT algorithm cannot be used as Doppler filter bank).

The effect of PRI staggering for use with STAP has been discussed in [6]. It was demonstrated that the optimum (LR) processor can cope well with staggered PRI, provided that the Doppler filters are matched to the staggered pulse sequence.

2.2. FIR Filtering with Staggered PRI

Now the question arises how an extremely efficient clutter filter technique such as the adaptive space-time FIR filter can operate with staggered PRI. Staggering the transmit pulses means that the received echo sequence is no longer stationary. Recall that the efficiency if the adaptive FIR relies on stationary data sequences.

Stationarity of the echo sequence means that the space-time submatrices ($m = 1, 2, 3$) in Figure 2 are equal. If the pulse sequence is staggered these matrices are different. A straight forward approach to cope with this non-stationarity is to readapt the filter at each PRI. That means, at each PRI the space-time submatrix is estimated anew. Then we obtain a space-time FIR filter with time-varying coefficients.

The adaptation of the FIR filter with each PRI causes additional expense in terms of computations. This is, however, tolerable, because the FIR filter is based on a small number of coefficients. Therefore, the associated time-depending subcovariance matrix is very small and needs only very few clutter echo samples for estimation. These can easily been taken at each PRI from the received range samples.

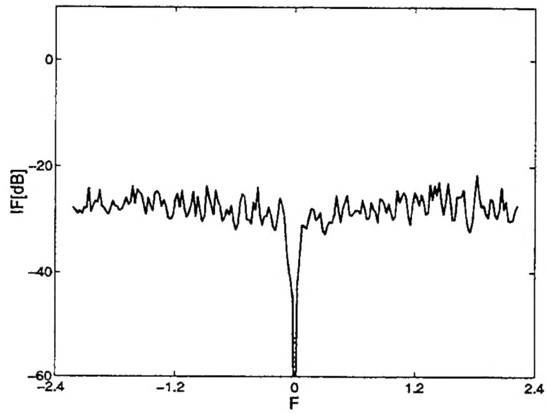

Figure 7: FIR filter, constant coefficients, randomly staggered PRI

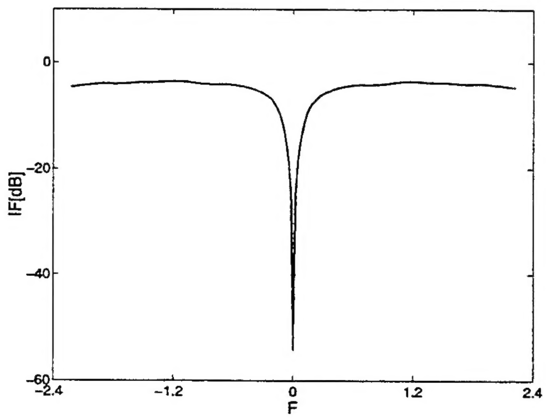

Figure 8: FIR filter, sliding calculation of coefficients, randomly staggered PRI

3. NUMERICAL EXAMPLES

The principle of clutter FIR filtering with time-varying coefficients is illustrated in Figures 3-8. As example, a sidelooking radar with linear array antenna was assumed. The look direction is perpendicular to the flight path, i.e., broadside.

Figure 3 shows the improvement factor in signal-to clutter+noise ratio versus the normalized target Doppler frequency. The PRF is constant and has been chosen so that ambiguous clutter notches show up in the clutter band (PRF= 4×Nyquist of the clutter band). The primary clutter notch is at F = 0, The other notches are repetitions due to Doppler ambiguity.

The same kind of IF plot has been calculated for the space-time FIR filter as given by Figure 4. The curve is quite similar to the one in Figure 3, however, with slightly broadened and deeper clutter notches. In Figure 5 we applied the principle of re-adaptation on echo data with constant PRF. Then the filter coefficients are calculated from the different submatrices ($m = 1 \ldots 3$). As can be noticed Figure 5 is identical to Figure 4. The reason is obvious: For constant PRF the echo sequence is stationary, and the submatrices are identical.

Let us now introduce a pseudorandom stagger code. Figure 6 shows again the behaviour of the optimum LR processor. As can be seen there is only one clutter notch left. The ambiguities do not show up anymore.

Applying a space-time FIR filter with constant coefficients leads to an IF curve as shown in Figure 7. There is only one clutter notch, however, due to the mismatch of the constant filter to the stagger pattern we obtain heavy losses in the passband of the filter. A FIR filter with sliding computation of the filter coefficients yields an IF curve as shown in Figure 8. We notice that except for a loss of a few dB a good filter characteristics is obtained.

4. CONCLUSIONS

Space-time least sqares FIR filters are a highly efficient way of clutter rection in air- or spaceborne radar. Radar operation with staggered PRI may be an attractive feature of airborne pulse-Doppler radars, with the potential of unambiguous Doppler estimation and avoidance of blind velocities. The optimum STAP processor as suggested by the likelihood ratio test can cope well with instationarities of the received echo sequence caused by PRI staggering. FIR filters with constant coefficients are by nature based on stationary echo sequences. Such filters, however, can be applied to staggered echo sequences if the filter is re-adapted with every PRI. It is shown by numerical examples that the time-varying space-time FIR filter can operate well on staggered echo data. The penalty for staggering is a loss in signal-to-clutter ratio of a few dB.

5. REFERENCES

[1] Burg, J. P., "Maximum Entropy Spectral Analysis", in: *Proc. NATO Advanced Study Institute*, Enschede, Netherlands, 1968

[2] Klemm, R., Space-Time Adaptive Processing - Principles and Applications (IEE, London, UK, 1998)

[3] Haimovich, A. L., Bar-Ness, Y., "An Eigenanalysis Interference Canceler", *IEEE Trans. Signal Processing*, Vol. 39, No. 1, January 1991, pp. 76-84

[4] Goldstein, J. S., Reed, I. S., "Theory of Partially Adaptive Radar", *IEEE Trans. AES*, Vol. 33, No. 4, October 1997, pp. 1309-1325

[5] Goldstein, J. S., Reed, I. S., Scharf, L ., "A multistage representation of the Wiener filter based on orthogonal

projections", *IEEE Trans. IT*, Vol. 44, No. 7, November 1998, pp. 2943-2959

[6] Klemm, R., "STAP with staggered PRF", *RADAR'99*, 17-21 May 1999, Brest, France

[7] Swindlehurst, A. L., Parker, P., "Parametric clutter rejection for space-time adaptive processing", *ASAP 2000 Workshop*, , MIT LL, Lexington, USA, 13-14 March 2000, pp. 7-12

[8] Klemm, R., Ender, J., "New Aspects of Airborne MTI", *Proc. IEEE Radar 90*, Arlington, USA, 1990, pp. 335-340

[9] Roman, J. R., Rangaswamy, M., Davis, D. W., Zhang, Q., Himed, B., Michels, J. H., "Reduced rank STAP performance analysis", *IEEE Transactions AES*, Vol. 36, No. 2, April 2000, pp. 677-692

[10] Baranoski, E. J., "Improved Pre-Doppler STAP Algorithm for Adaptive Nulling in Airborne Radars", *IEEE Proc. 29th ASILOMAR Conference on Signals, Systems and Computers*, Pacific Grove, USA, 30 Oct.-2 Nov. 1995, pp. 1173-1177

[11] Ward, J., "Space-Time Adaptive Processing for Airborne Radar", *Technical Report No. 1015*, Lincoln Laboratory, MIT, December 1994